

Anomalous-Hall Néel textures in altermagnetic materials

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Recently, the altermagnets, a new kind of collinear antiferromagnet with nearly zero net magnetization and momentum-dependent spin-splitting of bands, have sparked great interest. Despite simple magnetic structures, these altermagnets exhibit intriguing and intricate dependence of anomalous Hall effect (AHE) on the Néel vector, in contrast to the conventional perpendicular configuration of Hall current with magnetization in ferromagnets. However, the fundamental relationship between the AHE and the Néel vector remains largely elusive. Here, we reveal all the unconventional anomalous Hall textures in the Néel vector space, dubbed anomalous-Hall Néel textures (AHNTs) for altermagnets. Specifically, we identify 10 types across four categories of AHNTs for all altermagnets. Notably, we find that AHNTs resemble the known spin textures in momentum space and further reveal their symmetry origin. Meanwhile, we examine our key discoveries in prototypical altermagnets. Our work offers a thorough understanding of AHE in altermagnets and a complete and pictorial classification of altermagnets based on the geometry of response functions.

anomalous Hall effect, altermagnets, Néel vector, spin texture

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1 Introduction

The anomalous Hall effect (AHE) [1] and its optical counterpart magneto-optical effect (Kerr and Faraday effect) [2] are the salient manifestations of time-reversal symmetry breaking in magnetically ordered materials. The Hall current

$\mathbf{j}_H = \boldsymbol{\sigma}_H \times \mathbf{E}$ is determined by the anomalous Hall vector, $\boldsymbol{\sigma}_H = (\sigma_{Hx}, \sigma_{Hy}, \sigma_{Hz})$ and $\sigma_{Hi} = \varepsilon_{ijk} \sigma_{jk}^A$ ($i, j, k \in (x, y, z)$) is the anomalous Hall conductivity, which is intimately linked to the Berry curvature of electrons in momentum space [3,4]. In ferromagnets and ferrimagnets, $\boldsymbol{\sigma}_H$ is usually believed to be parallel to the magnetization $\mathbf{M}$, that is, the radial structure ($\boldsymbol{\sigma}_H \parallel \mathbf{M}$) in the magnetization space (such as Figure 1(a)), which leads to the conventional perpendicular configuration

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$\mathbf{j}_H \perp \mathbf{M}$. This long-time belief has been revisited by the recent progress of in-plane AHE [5-9]. Notably, AHE has been observed in nearly compensated noncollinear antiferromagnetic (AFM) materials [4,10-13]. Meanwhile, unconventional spin currents [14,15] and field-free magnetization switching [16,17] in Kagome nonlinear AFM (such as Mn_3Sn and Mn_3Ge) have been reported. Moreover, in these systems, the sum of the radial components $\sigma_H \parallel \mathbf{M}$ in the different spin sublattices is negligible; however, the non-radial components may exist and exhibit complicated behaviors.

Altermagnets [18-26], classified within the realm of collinear AFM materials, feature vanishing net magnetization constrained by symmetry, but exhibit momentum-dependent spin-split band structures akin to ferromagnets [27-33]. Remarkably, AHE [34-44] and the magneto-optical effects [45,46] have been observed in these materials, inspiring intensive investigations [47-53]. Despite the relatively simple magnetic structures in altermagnetic materials, the relationship between σ_H and the Néel vector $\mathbf{n}$ (the unit vector indicating the magnetic direction difference between two spin sublattices) unexpectedly exhibits rich and puzzling features [40,45,54,55], unlike the conventional radial structure ($\sigma_H \parallel \mathbf{M}$) in ferromagnets. However, a proper theory to build the fundamental relation between σ_H and the Néel vector $\mathbf{n}$ for all altermagnets remains elusive.

In this work, we treat the Néel vector as an extrinsic parameter to investigate anomalous Hall configurations in the Néel vector space $\sigma_H(\mathbf{n})$ for altermagnets, which can be dubbed as anomalous-Hall Néel textures (AHNTs). We further establish a complete classification of these AHNTs, and find AHNTs exhibit unusual relationships in analogy to the spin textures [56-58] in momentum space (Figure 1), such as Dresselhaus-like, Rashba-like, persistent, and pure cubic textures. Furthermore, we classify the altermagnetic materials according to the types of AHNTs and further verify our main results in several typical altermagnets.

2 Theoretical model

Since altermagnetic materials are all in symmetry-compensated collinear AFM states with nearly zero net spin magnetization, it is a natural way to analyze the AHE in terms of the Néel vector $\mathbf{n}$ [8,54,55], where $\mathbf{n}$ is a unit vector ($|\mathbf{n}| = 1$). To determine the profiles of $\sigma_H(\mathbf{n})$, we treat $\mathbf{n}$ as an extrinsic parameter [59] rather than an intrinsic structural parameter in the traditional magnetic group method (see Supporting Information Note 2) [60], and we named it as “extrinsic parameter” method. This approach enables us to analyze the $\sigma_H(\mathbf{n})$ relation in the framework of the nonmagnetic space

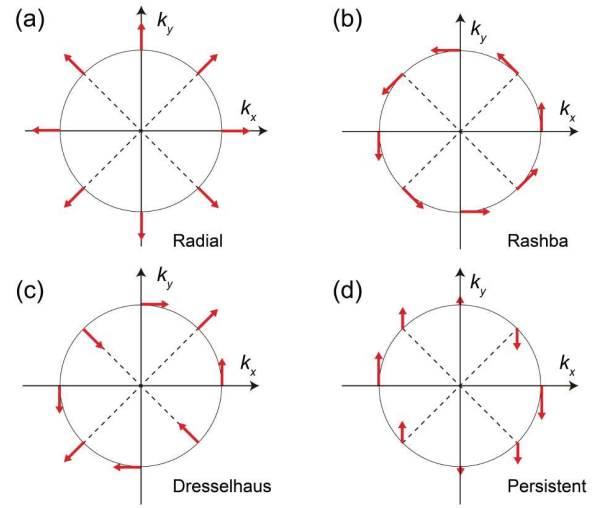


Figure 1 (Color online) Spin textures in the momentum space for nonmagnetic materials without inversion symmetry. (a) Radial $\beta_R(\sigma_x k_x + \sigma_y k_y)$; (b) Rashba $\lambda_R(k_x \sigma_y - k_y \sigma_x)$; (c) Dresselhaus $\lambda_D(k_x \sigma_y + k_y \sigma_x)$; (d) persistent $\lambda_R(k_x \sigma_y - k_y \sigma_x) + \lambda_D(k_x \sigma_y + k_y \sigma_x) = 2\lambda_R k_x \sigma_y$ with $\lambda_R = \lambda_D$ here. σ_x , σ_y , and σ_z are the Pauli matrices.

group. The space group operation $\{R | \mathbf{t}\}$ acting on $\mathbf{n}$ results in the change from $\sigma_H(\mathbf{n})$ to $\sigma_H(\{R | \mathbf{t}\}\mathbf{n})$, and it is equivalent to applying the point symmetry operation R directly on σ_H , i.e.,

$$\sigma_H(\{R | \mathbf{t}\}\mathbf{n}) = R\sigma_H, \quad (1)$$

where the translation operation $\mathbf{t}$ is omitted on the right side, as it does not impact on macroscopic physical properties. Using eq. (1), we can show that the antiferromagnets, in which two spin sublattices are connected either by inversion P or pure translation $\mathbf{t}_c/2$, do not permit the existence of the AHE, because these symmetries only reverse $\mathbf{n}$ not σ_H (Figure 2(a) and (b)). Coincidentally, the breaking of these two symmetries [20,21,23,24,61] leads to the band splitting without the spin-orbit coupling (SOC) effect, forming altermagnets. Notably, the spins are compensated, meaning the AHE in altermagnets is attributed to intrinsic orbital angular momentum [34].

Under mirror or glide mirror symmetry, the relationship between σ_H and $\mathbf{n}$ turns out to be non-trivial. As shown in Figure 2(c), when $\mathbf{n}$ is parallel to the glide mirror, the exchange of magnetic atoms with opposite spins preserves the Néel vector $\mathbf{n}_\parallel$ (see Supporting Information Note 1). However, the glide mirror reverses the sign of $\sigma_{H\parallel}$. Conversely, when the Néel vector is perpendicular to the glide mirror, the glide mirror operation reverses $\mathbf{n}_\perp$, while leaving the vertical component $\sigma_{H\perp}$ unchanged as it is an axial vector, which is shown in Figure 2(d). To ensure eq. (1) holds, σ_H must be perpendicular to $\mathbf{n}$ in the cases of Figure 2(c) and (d). Comparing to the general relationship $\sigma_H \parallel \mathbf{m}$ in ferromagnets, $\sigma_H \perp \mathbf{n}$ relation can emerge in altermagnets

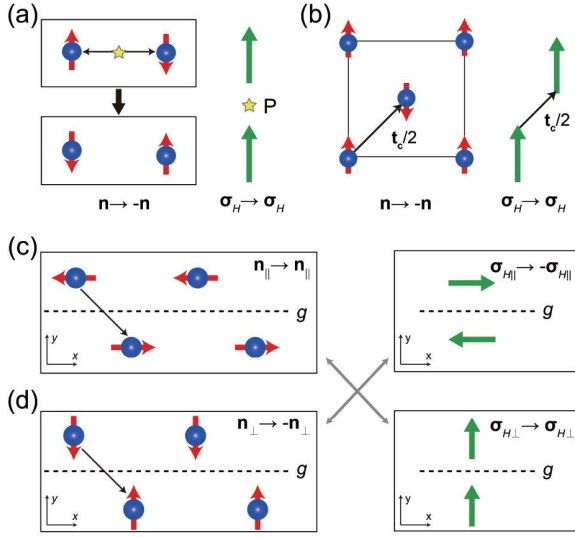


Figure 2 (Color online) Transformations of the Néel vector $\mathbf{n}$ and the anomalous Hall vector σ_H under the (a) inversion symmetry P , (b) $\mathbf{t}_c/2$ translation, and (c), (d) glide mirror g operations. The thin arrows between atoms denote the transformations of atom positions under the symmetry operations. The gray cross arrows between (c) and (d) denote the reasonable σ_H - $\mathbf{n}$ relationships in altermagnets.

[40,45,54,55].

If the directions of the magnetic moments change continuously while remaining strictly anti-parallel between the two spin sublattices, the Néel vector $\mathbf{n}$ traces out a surface resembling a Bloch sphere $|\mathbf{n}| = 1$. We can regard $\sigma_H(\mathbf{n})$ relation as the inner structure of σ_H in the Néel vector space. In Figure 2(c) and (d), we focus on two specific points in the Néel vector space: $\mathbf{n} = (1, 0, 0)$ and $\mathbf{n} = (0, 1, 0)$. Additional symmetries further lead to $\sigma_H(\mathbf{n})$ exhibit unconventional configurations like spin textures in the momentum space. In analogy to the spin texture [56–58], we thus refer to the $\sigma_H(\mathbf{n})$ relationship as the AHNT.

To gain analytical results of $\sigma_H(\mathbf{n})$, we expand σ_H as a Taylor polynomial with respect to $\mathbf{n}$,

$$\sigma_H(\mathbf{n}) = \mathbf{T}^{(2)} \cdot \mathbf{n} + \mathbf{T}^{(4)} : \mathbf{n}\mathbf{n}\mathbf{n} + \dots \quad (2)$$

The Taylor expansion is guaranteed because the SOC interaction acts as a perturbation in the AHE [8]. The Onsager reciprocal relation allows only the odd terms in eq. (2), due to its oddness under time-reversal operation. $\mathbf{T}^{(2)}$ and $\mathbf{T}^{(4)}$ are the two-order and four-order tensors, respectively. Alternatively, σ_H can be expressed in terms of components,

$$\sigma_{Hi} = \sum_{j=1}^3 T_{ij}^{(2)} n_j + \sum_{j,k,l=1}^3 T_{ijkl}^{(4)} n_j n_k n_l + \dots \quad (3)$$

The transformation of σ_H under the space group operation $\{R | \mathbf{t}\}$ in eq. (1) becomes

$$\{R | \mathbf{t}\} : \sigma_H \rightarrow \det(R) D(R) \sigma_H, \quad (4)$$

where $D(R)$ is the matrix of symmetry operation R , and $\det(R)$ is the determinant of $D(R)$. The transformation of the Néel vector $\mathbf{n}$ under $\{R | \mathbf{t}\}$ is given by

$$\{R | \mathbf{t}\} : \mathbf{n} \rightarrow \pm \det |R| D(R) \mathbf{n}, \quad (5)$$

where the $+$ and $-$ signs indicate whether the two magnetic sublattices with opposite spins are exchanged or not, respectively. Our method properly deals with the transformation of the Néel vector under the symmetry operations (see Supporting Information Note 2). Although the time-reversal symmetry operation is not explicitly involved in this method, the $\pm$ symbol in eq. (5) essentially reflects the switch of the magnetic moment under time-reversal operation.

With the aid of eqs. (1)–(5), we can obtain the symmetry-adapted tensors $\mathbf{T}^{(2)}$ and $\mathbf{T}^{(4)}$, and identify the invariant polynomials of $\mathbf{n}$ in eq. (3). Meanwhile, we have developed an automatically computational code to implement our method (see Supporting Information Note 2).

3 Results and discussions

3.1 Classification of AHNTs for altermagnets

With the above extrinsic parameter method, we can conduct a comprehensive study on the $\sigma_H(\mathbf{n})$ relations and corresponding AHNTs in the altermagnets. Since the $\mathbf{T}^{(2)}$ and $\mathbf{T}^{(4)}$ tensors in eq. (2) resemble the general axial tensors, it suffices to utilize the nonmagnetic group, rather than the magnetic group, to investigate the symmetry constraints imposed on $\mathbf{T}^{(2)}$ and $\mathbf{T}^{(4)}$ in the context of the “extrinsic parameter” method. We found 10 kinds of AHNTs in four categories for altermagnets, based on ten non-centrosymmetric and non-chiral point groups (see Supporting Information Note 4). We can see that the Rashba-like, Dresselhaus-like, and pure cubic AHNTs can exist in altermagnetic materials, while the radical texture, which is common in ferromagnetic materials, is absent.

As summarized in Table 1 [62,63], we find that the altermagnetic materials do not support the radial AHNTs as the ferromagnets due to the traceless characteristic of $\mathbf{T}^{(2)}$. If the symmetry composed on σ_H is equivalent to polar group $3m$, $6mm$, or $4mm$, only the Rashba-like AHNT exists. Besides, the pure Dresselhaus AHNTs can only occur if the symmetry composed on σ_H identifies with the non-polar tetragonal point group $\bar{4}$, $\bar{4}2m$, or $\bar{4}m2$. If altermagnetic materials lack higher-order rotational n or helical n_q symmetries (where $n \geq 3$), there are no constraints on the coefficients of the first-order Néel vector terms. The mixed AHNT containing both Rashba and Dresselhaus components hence appears (Table 1). Under symmetry constraints equivalent to monoclinic (m) or orthogonal ($mm2$) group, $\sigma_H(\mathbf{n})$ exhibits mixed Rashba and Dresselhaus AHNTs. Interestingly, when the symmetry associated with σ_H corresponds to $\bar{6}$, $\bar{6}m2$, or $\bar{4}3m$ point

Table 1 Ten kinds of AHNTs in four categories for altermagnetic materials. The candidate materials are sourced from the MAGNDATA database [62] and refs. [23,24,39,63]

AHNT	Expression and equivalent constrained point groups	Candidate materials
Rashba-like	$3m, 6mm, 4mm$ $\sigma_H = A(n_y, -n_x, 0)$	Trigonal: $\text{NiCO}_3, \text{MnCO}_3, \text{FeCO}_3, \text{FeF}_3, \text{CoF}_3, \alpha\text{-Fe}_2\text{O}_3$ Tetragonal: $\text{Ta}_2\text{FeB}_2, \text{KMnF}_3$
Dresselhaus-like	$\bar{4}m2: \sigma_H = A(n_y, n_x, 0)$ $\bar{4}2m: \sigma_H = B(n_x, -n_y, 0)$ $\sigma_H(\bar{4}) = \sigma_H(\bar{4}m2) + \sigma_H(\bar{4}2m)$	Tetragonal: $\text{RuO}_2, \text{CoF}_2, \text{MnF}_2, \text{NiF}_2, \text{LiFe}_2\text{F}_6, \text{ZrMn}_2\text{Ge}_4\text{O}_{12}$
Mixed texture	$m, mm2$ $\sigma_H = (An_y, Bn_x, 0)$ $A \neq \pm B$	Orthogonal: $\text{LaMnO}_3, \text{NdMnO}_3, \text{ScCrO}_3, \text{NaOsO}_3, \text{Cu}_2\text{V}_2\text{O}_7, \text{NiFePO}_5, \text{Mn}_2\text{SeO}_3\text{F}_2$
Cubic texture	$\bar{6}, \bar{6}m2 (\bar{6}2m), \text{ or } \bar{4}3m$ $\sigma_H = \mathbf{T}^{(4)} : \mathbf{mnn}$	Hexagonal: $\text{MnTe}, \text{CrSb}, \text{Ba}_3\text{NiRu}_2\text{O}_9, \text{CrNb}_4\text{S}_8$

groups, there is no first-order term for $\sigma_H(\mathbf{n})$, resulting in cubic AHNTs as the lowest order in altermagnets. Thus, our classification enables the identification of all AHE behaviors in altermagnets and facilitates the efficient discovery of corresponding materials. Notably, we find a striking similarity between AHNTs and spin textures in momentum space [58,64] (see Supporting Information Note 3).

It should be noted that, similar to the Berry curvature multipoles in momentum space [65–67] and the multipoles of spin in real space [43,68–70], eq. (2) can be viewed as the AHE multipoles in the Néel vector space. In Supporting Information Note 5, we project σ_H to real sphere harmonics and get the corresponding multipoles in the Néel vector space. The multipoles of σ_H in the Néel vector space reflect the complexity of the SOC effect on the AHE [8].

3.2 Origin of similarity between AHNTs and spin textures

To uncover the symmetry origin of the similarity between AHNTs and spin textures, we consider the general SOC Hamiltonian as $H_{\text{SOC}} = \mathbf{\Omega}(\mathbf{k}) \cdot \boldsymbol{\sigma}$, where $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices, and $\mathbf{\Omega}(\mathbf{k})$ is a wave-vector dependent spin-orbit field [58], which governs the spin expectation at each $\mathbf{k}$ -point. $\mathbf{\Omega}(\mathbf{k})$ behaves as an axial vector, as the AHE conductivity vector σ_H ; whereas, the Néel vector $\mathbf{n}$ is a polar-like vector as the momentum $\mathbf{k}$, because it switches its sign under the inversion symmetry (see Figure 2(a)).

The two similarities result in the fact that point group operations impose equivalent constraints on their forms of corresponding expanded coefficients. For a first-order Taylor expansion, $\mathbf{\Omega}(\mathbf{k})$ can be expressed as

$$\Omega_i(\mathbf{k}) = \sum_{j=1}^3 \mathcal{T}_{ij}^{(2)} k_j, \quad (6)$$

where the coefficient matrix $\mathcal{T}_{ij}^{(2)}$ is an axial tensor, similar to $\mathbf{T}^{(2)}$ in eq. (2). For example, under the polar $3m$, $4mm$, and $6mm$ point groups [64], only two nonzero elements of $\mathcal{T}^{(2)}$

exist: $\mathcal{T}_{xy} = -\mathcal{T}_{yx}$. We can obtain $\mathbf{\Omega}(\mathbf{k}) = \mathcal{T}_{xy}(k_y, -k_x, 0)$ and $H_{\text{SOC}} = \mathcal{T}_{xy}(k_y\sigma_x - k_x\sigma_y)$, leading to the Rashba-type spin texture $\mathbf{s}(\mathbf{k}) \left(\mathbf{s} = \frac{\hbar}{2} \left\langle \boldsymbol{\sigma} \left| \boldsymbol{\varphi} \right| \boldsymbol{\varphi} \right\rangle \right)$, as shown in Figure 1(b).

The configurations of $\mathbf{T}^{(2)}$ tensors align with $\mathcal{T}^{(2)}$ tensors across all point groups, as given in Supporting Information Note 3.

Besides, due to the time-reversal symmetry, the spin texture satisfies $\mathbf{s}(\mathbf{k}) = -\mathbf{s}(-\mathbf{k})$ in nonmagnetic materials. A similar relation also exists in AHNTs: $\sigma_H(\mathbf{n}) = -\sigma_H(-\mathbf{n})$, due to the odd parity of σ_H with the time-reversal operation. Consequently, the categories of AHNTs listed in Table 1 bear a resemblance to the spin texture.

Our theory well reproduces the $\sigma_H(\mathbf{n})$ relation of rutile-type (such as RuO_2 [31, 34, 35, 71]) and NiAs-type ($\alpha\text{-MnTe}$ [28,40]) altermagnetic materials with Dresselhaus-like and pure cubic AHNTs, respectively (see Supporting Information Notes 6 and 7). The predicted AHNTs are consistent with the AHE [35,40] and X-ray magnetic circular dichroism (XMCD) [72,73] experiments. Except for the two widely studied altermagnets, we introduce two previously unidentified AHNTs in the following.

3.3 Calcite-type altermagnets

Altermagnetic materials NiCO_3 , MnCO_3 , FeCO_3 , and FeBO_3 have the calcite structure with $R\bar{3}c$ (No. 167) space group. In this structure, the magnetic atoms occupy the 6b positions at $(0, 0, 0)$ and $(1/2, 1/2, 1/2)$, and they are bonded to six equivalent O atoms, forming corner-sharing octahedra, as illustrated in Figure 3(a). The inversion center is located on each magnetic atom (not the midway point between them), breaking PT symmetry and enabling band splitting even without SOC effect. Unlike the 4_2 screw symmetry in the rutile-type altermagnets, the three-fold rotational symmetry present here does not switch the opposite-spin magnetic atoms, besides the glide mirror operations interchange them.

Consequently, the symmetry composed of σ_H is equivalent to $3m1$. According to the above symmetry characteristics, we obtain the corresponding $\sigma_H(\mathbf{n})$ (See Supporting Information Note 6.3),

$$\sigma_H(\mathbf{n}) = (An_y, -An_x, 0) + O(n^3). \quad (7)$$

Our numerical results for the effective tight-binding (TB) model are consistent with eq. (7), as shown in Figure 3(b). Unlike the rutile-type altermagnets, the m_z mirror symmetry does not exist in calcite altermagnets, which allows σ_{Hz} when $\mathbf{n}$ is in the xOy plane and exhibits a third-order contribution $\sigma_{Hz} \sim n_x(n_x^2 - 3n_y^2)$. Although, σ_H in the xOy plane for the rutile-type and calcite-type altermagnets exhibit similar trigonometric characteristics, essential differences leading to calcite-type altermagnets hold a Rashba-like AHNT when the third order terms are omitted, while the AHNT of rutile-type altermagnets obeys a Dresselhaus-like configuration. This Rashba-like texture in calcite-type altermagnets ensures that the vector σ_H is parallel to the latitude plane and tangent to the latitude circles, as shown in Figure 3(c).

In addition to calcite-type altermagnetic materials, altermagnets with trigonal structures such as CaLiOsO_6 , $\text{Ca}_3\text{LiRuO}_6$, $\text{Sr}_3\text{LiRuO}_6$, CoF_3 , and FeF_3 also exhibit $R\bar{3}c$ symmetry, giving rise to a similar $\sigma_H(\mathbf{n})$ relation. The XMCD [74] and AHE [75] experiments in $\alpha\text{-Fe}_2\text{O}_3$ show the $\sigma_H \perp \mathbf{n}$ relations when the Néel vector is along the in-plane crystallographic direction and σ_H vanishes when it is along the

[001] direction, suggesting a possible Rashba-like AHNT. In fact, the Rashba-type AHNTs are not limited to trigonal altermagnets. For instance, the tetragonal KMnF_3 with the $I4/mcm$ (No. 140) space group and Ta_2FeB_2 [39] with the $P4/mbm$ (No. 127) space group can also display Rashba-like AHNTs, as given in Table 1.

3.4 Perovskite-type altermagnets

The perovskite-type altermagnetic materials represent a broad class of AFM materials in the formula of ABO_3 , belonging to the space group $Pnma$ (No. 62). An octahedron formed by six oxygen atoms surrounds each magnetic atom B, with non-magnetic A atoms situated at the center of these octahedra, as shown in Figure 4(a). Typically, the magnetic atoms B occupy the 4b Wyckoff positions at $(1/2, 0, 0)$, $(0, 1/2, 1/2)$, $(1/2, 1/2, 0)$, and $(0, 0, 1/2)$. Consequently, there are 3 ($C_4^2 C_2^2 / A_2^2 = 3$) kinds of collinear AFM configurations, indicated by A-type (e.g., LaMnO_3 , PrMnO_3 , NdMnO_3), C-type (e.g., CaCrO_3 [38], ScCrO_3), and G-type (e.g., NaOsO_3 , LaCrO_3 , TbCrO_3 , NdFeO_3 , TbFeO_3 , etc.), as illustrated in Figure 4(b).

Although the symmetry restrictions on σ_H are the same in the three magnetic configurations, the constraints on the Néel vector $\mathbf{n}$ turn out to be different (see Supporting Information Note 7.4). Therefore, the relations of $\sigma_H(\mathbf{n})$ up to the third-order differ in the three magnetic configurations:

$$\begin{cases} \sigma_H^A(\mathbf{n}) = (An_y, Bn_x, 0) + (an_x^2 n_y + bn_y^3 + cn_y n_z^2, dn_x^3 + en_x n_y^2 + fn_x n_z^2, gn_x n_y n_z), \\ \sigma_H^C(\mathbf{n}) = (An_z, 0, Bn_x) + (an_x^2 n_z + bn_y^2 n_z + cn_z^3, dn_x n_y n_z, en_x^3 + fn_x n_y^2 + gn_x n_z^2), \\ \sigma_H^G(\mathbf{n}) = (0, An_z, Bn_y) + (an_x n_y n_z, bn_x^2 n_z + cn_y^2 n_z + dn_z^3, en_x^2 n_y + fn_y^3 + gn_y n_z^2). \end{cases} \quad (8)$$

The diverse $\sigma_H(\mathbf{n})$ relation is helpful for determining the magnetic structures of perovskite-type altermagnets. It is noteworthy that if the mirror or glide mirror operations do not switch the magnetic atoms with opposite spins (as shown

in Figure 4(b)), $\sigma_H(\mathbf{n})$ is vanished when the Néel vector $\mathbf{n}$ is perpendicular to the mirror or glide mirror planes, according to eq. (8). The first order $\sigma_H(\mathbf{n})$ relationship exhibits the mixed Rashba and Dresselhaus AHNTs, because no sym-

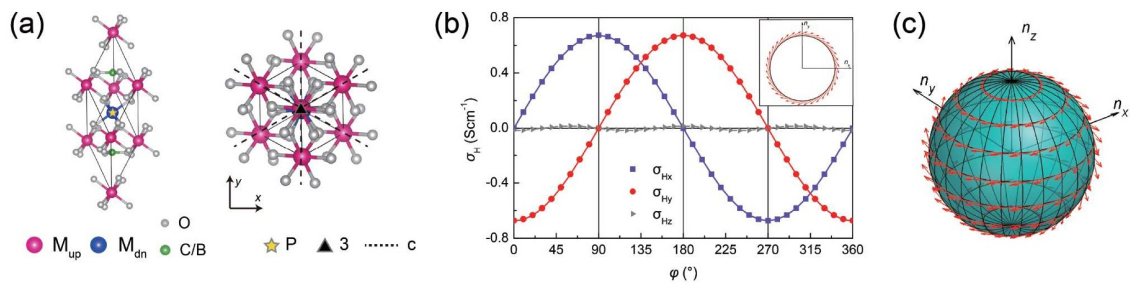


Figure 3 (Color online) (a) Crystal structure and symmetry of calcite-type altermagnets. (b) Corresponding relations of $\sigma_H(\mathbf{n})$ in the xOy plane and (c) AHNT in the whole Néel vector space. The inset in (b) is the Rashba-like AHNT in the xOy plane. The points in (b) are the numerical data calculated by the effective TB model (see Supporting Information Note 6.3), and the curves are the analytical $\sigma_H(\mathbf{n})$ function fitted by eq. (7). The parameters of the TB model are in Supporting Information Note 7.2.

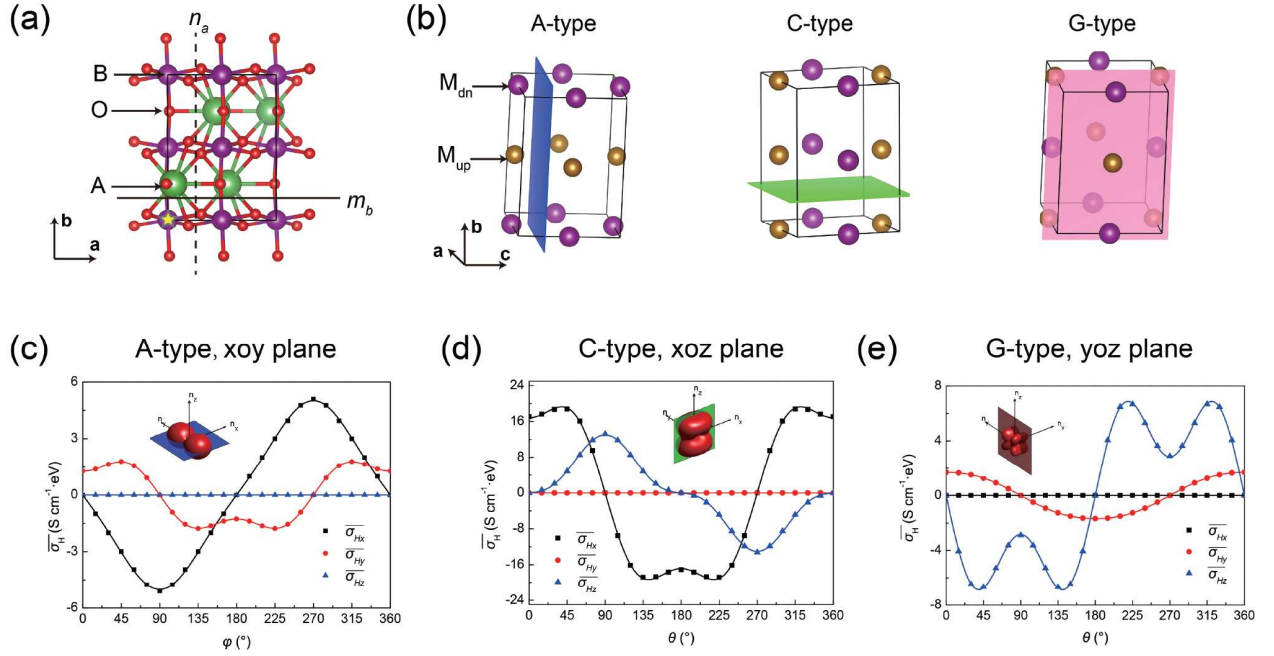


Figure 4 (Color online) (a) Crystal structure of perovskite-type altermagnetic materials ABO₃. (b) Magnetic structure of A-, C- and G-type AFM configurations. The magnetic atoms with different spins are represented with different colors. The colored planes indicate that the mirror or glide mirror operations do not switch the magnetic atoms with opposite spins. (c)-(e) Angle dependent of spectral integrals of optical Hall conductivity $\bar{\sigma}_H$ for A-, C-, and G-type perovskite-type altermagnetic materials, respectively. The solid lines are the fitting curves by eq. (8). The points in (c)-(e) are the numerical data calculated by the effective TB model (see Supporting Information Note 6.4), and the curves are the analytical $\bar{\sigma}_H(\mathbf{n})$ function fitted by eq. (7). The insets are the $Abs|\bar{\sigma}_H|$ profiles in the Néel vector spaces. The parameters of the TB model are in Supporting Information Note 7.4.

metry forces $A = B$ or $A = -B$.

Since most perovskite-type altermagnetic materials are insulators, we investigate the optical Hall conductivity of A-, C-, and G-type configurations. The calculated results using the TB model are consistent with eq. (8), as given in Figure 4 (c) and (d), where the spectral integrals of optical Hall conductivity $\bar{\sigma}_H = \int_{0+}^{\infty} \sigma_H(\omega) d\omega$ [12,45]. Previous experiments show that some perovskite antiferromagnets [76,77] exhibit the magneto-optical effect, while the $\sigma_H(\mathbf{n})$ relationship in them needs further experimental investigation.

4 Conclusions

The extrinsic parameter method is not only applicable to AHE but can also be extended to other magnetically related physical effects [78], such as the second harmonic generation effect [79], the spin Hall effect [80,81], and the nonlinear Hall effect [82-84].

Besides, AHNTs shed light on experimentally detecting the Néel vector in altermagnetic materials via the anomalous Hall transport (See Supporting Information Note 9). Importantly, we can utilize the characteristic that the σ_H is usually perpendicular to the Néel vector to design a vertical

magneto-optical isolator (see Supporting Information Note 9).

In summary, we treat the Néel vector as an extrinsic parameter to unveil the relationship between AHE and Néel vector in altermagnets. We present a comprehensive classification of AHNTs for altermagnets, and find a variety of unconventional AHNTs, including Rashba, Dresselhaus, persistent, and pure cubic AHE textures, which are absent in ferromagnetic materials. This complete framework of AHNTs serves as a pictorial category scheme for altermagnets. We found an intriguing similarity between the AHNTs in altermagnets and the spin textures in momentum space, and further elucidated their symmetry origin. Our findings provide a thorough analysis of AHE in altermagnets, and useful guidance for the detection of Néel vectors through AHE, and designing electronic and optoelectronic devices based on altermagnetic materials.

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Conflict of interest The authors declare that they have no conflict of interest.

Supporting Information

The supporting information is available online at <http://phys.scichina.com> and <https://link.springer.com>. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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